A Yield Model for Unthinned Sitka spruce (*Picea sitchensis* (Bong.) Carr) Plantations in Ireland

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SUMMARY

Over the past few decades the construction of yield models has progressed from the graphical through mathematical and biomathematic approach. The development of a biomathematical growth model for Sitka spruce plantations is described. It is suggested that this technique can serve as a basis for general yield model construction of plantation species in Ireland.

INTRODUCTION

The terms 'yield model' and 'yield table' are synonymous although the former reflects the greater flexibility in construction and application which is offered by computers. Yield models are applied in a general sense to presentations of expected growth of forest stands based on growth measured or inferred. They consist of variables such as volume, basal area, height, dbh and number of stems per unit land area for stands of various densities and ages on sites of different productive capacities.

Yield models are used in forestry for production forecasting, for yield control, for valuation and for the evaluation of economic alternatives or management strategies (Hamilton and Christie 1973). They also provide guidelines for solving many forest management problems and for testing various alternatives in decision making at local, regional or national levels. Yield models are currently in use, such as the yield models for forest management

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IRISH FORESTRY, 1982, Vol. 39, No. 2: 75-93.

by Edwards and Christie (1981) which replaced the British Forestry Commission Forest Management Tables (Hamilton and Christie 1971).

In the Republic of Ireland no yield tables have been produced for the management of Sitka spruce plantations. The Forest and Wildlife Service of the Department of Fisheries and Forestry uses BFC Management Tables for the management of these plantations. These tables are now regarded by many forest managers as being inadequate for Irish conditions (Gallagher 1972). Kilpatrick (1978) has also reported that unthinned Sitka spruce stands in Northern Ireland have higher basal area growth rate than is indicated in the BFC Yield Tables. This inadequacy of BFC Yield Tables for the management of Sitka spruce plantations in Ireland has been accentuated by the recent advances in silviculture such as site amelioration, increased planting espacement, respacing or no thinning regimes. Thus, yield models are urgently required in Ireland for the intensive management of plantations.

This paper presents a method based on Richards' Growth Function which has been developed for the construction of yield models for unthinned S.S. plantations of different initial espacements throughout the country.

Development of Yield Model Construction

Yield tables were used in Europe before the end of the eighteenth century; the first yield table is said to be produced by Hennert in 1791 (Gallagher 1965). Since the idea of yield table construction was first conceived, its basic functions remain the same. Major developments, however, have occurred in the construction over the years.

Foresters in the 18th and 19th centuries assembled their data in the form of tables which were based on graphically produced relationships between crop characteristics. This procedure was, however, found to be rather subjective so forest scientists started to look for mathematical expressions to depict the growth of forest trees. The qualities of mathematics (precision, predictive ability, abstract in decuctive logic or argument, medium of communications unaffected by barriers of human language) backed by the advent of electronic computer facilities influenced the shift from the graphical and tabular technique of yield model construction to the mathematical models of the 1950s and 1960s. During this period when scientifically designed growth studies became operative, various regression techniques were used. The conventional yield models obtained by these regression procedures are of limited use however because their predictive capability of growth does not take into account the biological complexities of tree growth, silvicultural practices and environmental factors. Therefore, these models are now being replaced by improved growth and yield functions.

GROWTH FUNCTIONS

Prodan (1968) discusses fully the many attempts by forest scientists to establish growth and yield functions for the growth prediction of forest trees. Examples of these functions are the exponential, allometric, Backmans etc. All of these are deterministic functions where given sets of conditions yield expected definite results. Stochastic models which are functional equations, where random variables are introduced were also tried by foresters. The Markov chain, Weibull distribution, logistic, Langevins etc., are some of the frequently tried stochastic functions for forest tree growth studies. These deterministic and stochastic functions still do not adequately describe the growth processes in forest trees because the biological complexities of tree growth are not taken into full account.

Pienaar and Turnbull (1973) pioneered a systematic approach of combining the biology of forest tree growth with mathematics. Their work was based on the research results of Von Bertalanffy (1951, 1957) and Richards (1959). The work of these two is well documented in the literature and is also detailed by Pienaar and Turnbull (1973). Because of its applicability to the study of tree growth, Richards' growth function is used to generate mathematical expressions of biological growth processes for Sitka spruce plantations in the Republic of Ireland.

DATA COLLECTION

The data used for this study were obtained from three sources within the country:

(a) From a survey of Sitka spruce plantations over the age of ten years carried out by the personnel of the Forest and Wildlife Service in 1966. In all, 198 temporary plots were measured in 16 countries covering over 36 forests (Gallagher 1972).

(b) From controlled thinning trials and spacing experiments established by Forest and Wildlife Service, Research Branch. Age of the plots range from 12 to 21 years. These data covered 13 counties in 27 forests. Lynch (1980) gives a comprehensive outline of these experiments.

(c) From two espacement trials at Drumhierney Plantation, Co. Leitrim. This plantation is owned by Mr. A. O'Rahilly and the experiment was established in collaboration with Professor Clear of University College, Dublin, in 1954. Various measuremets have been taken periodically since 1970.

In all cases, trees 7cm dbh and above were girthed with steel tape. A number of dominant trees were measured for top heights using the Blume Leiss hypsometer. Volumes of sample trees were computed from sectional measurements. The number of surviving stems, basal area, mean dbh and volume per plot were calculated. These were later converted to a per hectare basis.

Model Construction

Growth and yield studies over the years have led to the collection of much empirical evidence of the growth processes in relation to forest crops. This has been consolidated into theories or so called 'laws' which have formed the basis of yield model construction. Examples are Möller's theory of 1954 and Eichhorn's Law of 1901 (Assmann 1970) which state respectively that the volume increment of a stand does not vary over a wide range of stocking levels and that the total crop yield without exception is a function of height. As more information became available, those relations have been modified and refined to accommodate new data. An example of this is the inclusion of production classes as subdivisions of yield classes to cater for the fact that the relationship between cumulative volume production and top height is semi-stochastic rather than functional and varies to some extent with site conditions. Nonetheless, these original 'laws' have stood the test of time and continue to serve as an essential starting point of model construction particularly when data are limited or not available over the range required for detailed analysis. In the absence of adequate date, recourse must be made to growth 'laws' such as those earlier mentioned. This study is no exception and considerable use is made of two well established relationships:

1. Top height in relation to age is a good indicator of site productivity.
2. Top height is not significantly affected by stand density.

In constructing the growth model the first step is to relate top height to age irrespective of stocking density.

Therefore, all the top heights of the available data irrespective of their sources and espacements are plotted on age. By visual inspection, those top height-age points that follow a pattern in ascending order of growth are joined together. Thus, abstract growth series are obtained since growth has not been measured directly but is estimated from the differences between plots of various ages. This is the usual procedure in growth model construction when permanent sample plot data are not available.

Research workers are unanimously agreed that spacing affects dbh, basal area, volume and mortality. In order to quantify the effects of spacing upon these stand characteristics, the data were sorted into square spacing categories (1.6, 1.8, 2.4 and 4.0 meter) and the growth characteristics (vol/ha, ba/ha and mean dbh/ha) for each espacement were then plotted against top height as independent variable.

The growth model chosen for use in this study is based on the Von Bertalanffy growth function (1951, 1957) as modified by Richards (1959).

In its modified form the equation is:

$$W = A(1-be^{-kt})^{1/1-m}$$

where W = dependent variable (e.g. vol/ha)

t = independent variable (e.g. top height)

A = Asymptotic value of the growth characteristic under investigation (e.g. the maximum expected value of, say, vol/ha).

b = constant which is biologically unimportant.

k = rate constant which determines the average steepness of growth curve along the x-axis.

m = coefficient responsible for location of point of inflection on the curve and exclusively determines the shape of the curve.

e = base of Naperian logarithm.

The method used to fit the data to the Richards' function is that of Stevens (1951). Stevens' method solves the equation $Y=A+B.R^x$ by the method of least squares and determines the parameters A, B and R. This involves replacing $W^{(1-m)}$ by 'Y', $A^{(1-m)}$ by 'A', $-A^{(1-m)}$. b by 'B' and e^{-k} by 'R' respectively to present the Richards' function in the same form as that of Stevens.

Because of the detailed computational procedures involved in fitting Richards' function to data by the Stevens method, a computer package consisting of four programmes was developed for this study. The programmes used are as follows:

1. RICHAD.FOR which consists of a main programme and three principal sub-programmes.
2. BMD programme. (Biomedical Computer Programs, California).
3. CURVE.FOR which consists of a main programme and three sub-programmes.

YIELD.FOR which is made up of a main programme and a sub-programme.

Using the available data the first programme generates a number of growth equations for the establishment of relationships between top height and age over the range of site types as shown in Table 1[1]. The programme is then used to establish relationships between each of the crop characteristics vol/ha, ba/ha and mean dbh/ha and top height for each of the four espacements under investigation (Tables 2, 3 and 4). In this way growth models for the different growth characteristics for each espacement are obtained. The computer outputs which are generated are stored on files which are readily accessible to the other two computer programmes.

The second programme performs asymptotic regression analyses of the form $Y=P+QR^x$ (1)

where Y=Asymptotic 'A' values for the growth characteristics generated by Richards Growth Function Programme.

X=Common log of stand density.

P, Q, R=Parameters estimated by the programme.

In this study, the 'A' parameters of each growth characteristic (vol/ha, ba/ha, mean dbh) for the different stocking densities are regressed on the common logarithm of their initial stocking densities. Tables 5 and 6 show the input data and computer output respectively for the different growth characteristics.

The third programme processes the parameters. 'b', 'k' and 'm' generated by the first programme to give third degree polynomial equations for each parameter and spacing. Each parameter of the growth characteristic is regressed on the common logarithm of the stocking densities. Tables 7 and 8 show the input and output data respectively for the various growth characteristics and parameters.

'Adjusted' values of the Richards function growth parameter 'A' in relation to each planting espacement are obtained for the various

1 For all tables see pages 84 to 93.

growth characteristics using the equations generated by the asymptotic regression programme. Similarly, 'adjusted' values of the parameters 'b', 'k' and 'm' were obtained from the polynomial regression equations. Thus, for any given espacement a unique set of 'A', 'b', 'k' and 'm' parameters for each crop characteristic are calculated. When these values were inserted into the Richards' function, $W=A(1-be^{-kt})^{1/(1-m)}$, tabular values for each crop characteristic, for each espacement, were obtained. Thus, a single equation (Table 9) for each growth characteristic is generated and this single equation may be regarded as the biomathematical growth equation for Sitka spruce in Ireland. This biomathematical growth equation permits interpolation of values for those parameters where no data are available. Extrapolation beyond the range of data is also possible.

YIELD TABLE CONSTRUCTION FROM THE BIOMATHEMATICAL GROWTH MODELS

The fourth computer programme developed for this study consists of a main programme and a sub-programme called 'Forest'. 'Forest' uses the asymptotic and polynomial regression equations to generate 'adjusted' values of the parameters 'A', 'b', 'k' and 'm'. These adjusted values are inserted into the Richards' Growth Function, $W=A(1-be^{-kt})^{1/(1-m)}$, to generate yield table values. Compilation of the actual tables is done by the main programme. This follows the orthodox procedure of first relating top height to age and then relating the various growth characteristics to top height over the range of stocking densities.

To illustrate this procedure an example may be taken of a stand with a stocking density (variable X) of 3,000 stems per hectare (Table 10). The common logarithm of this number (3.4771) is substituted for the variable X in the biomathematical growth equations by means of sub-programme 'Forest' and the 'adjusted' parameters 'A', 'b', 'k' and 'm' are generated for each crop characteristic (Table 11). For a selected asymptotic height category the main programme computes top heights for given ages and then calculates the values for the different growth characteristics associated with each top height. The main programme also directs the printing of the values in tabular format (Table 12). For the asymptotic height category selected, a series of unique tables can be generated by altering the initial spacing (variable X) within the range 600 to 5,000 stems per hectare.

VALIDATION OF THE MODEL

In the validation procedure the values computed by the Richards' function compared with the original field data using a 't' test analysis. A one-way analysis of variance was also used for the same purpose but with three sets of data (Original, Richards', Biomathematical). The results indicated that there were no significant differences (95% level) between the original data and the data generated by the model.

DISCUSSION

Mathematical fitting of growth curves is a common practice in modern forest growth investigations. In the past, most of the functions employed were essentially artificial because the basis of these functions (linear, parabolic, logarithmic) were usually algebraic or geometric (Turnbull, 1963). In consequence they could only represent portions of growth curves. Biomathematical growth equations however have a biological interpretation and therefore can describe total growth curves.

In this study, the Richards' Growth Function was used in conjunction with data available to generate yield models for unthinned Sitka spruce plantations in Ireland. The potential of this method of yield model construction is only limited by the number of initial stocking densities and height categories selected. Since the original data available for Sitka spruce plantations of different espacements in Ireland were extremely limited the tables produced can be considerably refined as more data become available. Nonetheless, the statistical analyses carried out showed that the tables generated by this process were accurate. This high degree of accuracy was attributed to the fact that the 'm' values generated by the computer programme, although varying greatly among and within growth characteristics and espacements, were always less than unity. This ensured that the original significance of the function was maintained.

The models given in this paper are directly applicable only to unthinned Sitka spruce plantations in Ireland with stocking densities ranging from 600-5,000 stems per hectare. However, the growth rate in a thinned stand is identical with that of an unthinned stand of the same age and same basal area for a wide range of thinning regimes (Pienaar and Turnbull 1973). Thus, the models developed should also be applicable to thinned Sitka spruce stands. Similarly, where plantations are respaced before the onset of competition it should be possible to use the developed models.

This study has demonstrated that the effect of initial espacement on the growth of unthinned Sitka spruce stands can be adequately depicted by the Richards Growth Function. This should greatly facilitate research work on the growth of other plantation species in Ireland. There are indications from this work that volume production in Ireland is higher than in Britain for the same top height. However, more growth data are necessary over a wide range of sites to quantify this conclusively.

It is hoped that the result of this study will greatly enhance further research in the use of Richards' Growth Model for the study of growth and yield of man-made forests.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the assistance of the Forest and Wildlife Service in providing data for this study. The invaluable help of Miss Loretta O'Connor and the staff of the Computer Centre, University College, Dublin, is also acknowledged.

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TABLE 1

Value of the Parameters A, b, k and m
for Richards Function $W = A(1-be^{-kt})^{1/(1-m)}$
for Top Height and Age.

A	b	k	m
26.4767	1.0000	0.0319	0.5300
30.2367	1.0000	0.0306	0.4900
40.6958	1.0000	0.0197	0.3100
39.8395	1.0000	0.0243	0.3500
37.7746	1.0000	0.0351	0.5000
39.0250	1.0000	0.0344	0.4500
40.3514	1.0000	0.0362	0.4500
41.9092	1.0000	0.0375	0.4500
42.4380	1.0000	0.0413	0.4800
42.8787	1.0000	0.0459	0.5200
43.4904	1.0000	0.0547	0.5600
46.5831	1.0000	0.0447	0.4900

TABLE 2

Value of the Parameters A, b, k and m
for Richards Growth Function $W = A(1-be^{-kt})^{1/(1-m)}$
for the relationship between Volume Per Hectare and Top Height
for different spacings.

Spacing	Richards growth function parameters			
	A	b	k	m
1.6m	2002.5703	0.8506	0.0574	0.7100
1.8m	1873.0627	0.7375	0.0648	0.7900
2.4m	1640.1812	0.9590	0.0745	0.7700
4.0m	1602.7793	0.8851	0.0652	0.8000

TABLE 3

Value of the Parameters A, b, k and m
for Richards Function $W=A(1-be^{-kt})^{1/(1-m)}$
for the relationship between Basal Area Per Hectare
and Top Height for different spacings.

Spacing	Richards function growth parameters			
	A	b	k	m
1.6m	97.6943	0.6411	0.0785	0.4600
1.8m	84.8862	0.6755	0.1375	0.7500
2.4m	82.7402	1.0000	0.0878	0.4700
4.0m	71.0600	1.0000	0.1059	0.6900

TABLE 4

Value of the Parameters A, b, k and m
for Richards Function $W=A(1-be^{-kt})^{1/(1-m)}$
for the relationship between Mean DBH and Top Height
for different spacings.

Spacing	Richards function growth parameters			
	A	b	k	m
1.6m	38.1838	0.5206	0.0686	0.7500
1.8m	52.4082	0.6419	0.0424	0.6100
2.4m	51.4572	0.5198	0.0464	0.6500
4.0m	52.7102	0.3569	0.0662	0.8000

TABLE 5

Value of the 'A' Parameter calculated by the RICHAD. FOR. of each Growth Characteristic for four different spacings (Crop Density). (Input for BMD Programme).

Growth Characteristic	Log_{10} density	'A' values
Volume/ha	3.5642	2002.5703
	3.4771	1873.0627
	3.2388	1640.1812
	2.7959	1602.7793
BA/ha	3.5642	97.6943
	3.4771	84.8862
	3.2388	82.7402
	2.7959	71.0600
Mean DBH	3.5642	38.1838
	3.4771	52.4082
	3.2388	51.4527
	2.7959	52.7162

(Note: The above log densities represent spacing of 1.6, 1.8, 2.4 and 4.0 metres).

TABLE 6

Value of the Parameters P, Q and R of the Asymptotic Regression Equation $Y=P+QR^x$ for each growth characteristic calculated by BMD Programme. (Y='A' values from Richards Function and X=Log Density.)

Growth Characteristic	Growth Parameter	Final Estimates
Volume/ha	P	1585.5625
	Q	0.000002
	R	203.90686
Basal area/ha	P	68.5115
	Q	0.0020
	R	14.2892
Mean DBH/ha	P	–56.0625
	Q	151.1494
	R	0.8942

TABLE 7

Values for the b, k, and m Parameters of Richards Function for four different spacings (density) of each Growth Characteristic used as input for CURVE. FOR. Programme.

Growth Characteristics		Log Density	Growth Parameters b	k	m
Vol/ha	1	3.5642	0.8506	0.0574	0.7100
	2	3.4771	0.7375	0.0648	0.7900
	3	3.2388	0.9590	0.0745	0.7700
	4	2.7959	0.8851	0.0652	0.8000
Ba/ha	1	3.5642	0.6411	0.07850	0.4600
	2	3.4771	0.6755	0.1375	0.7500
	3	3.2388	1.0000	0.0878	0.4700
	4	2.7959	1.0000	0.10590	0.6900
Mean DBH	1	3.5642	0.5206	0.0686	0.7500
	2	3.4771	0.6419	0.0424	0.6100
	3	3.2388	0.5198	0.0464	0.6500
	4	2.7959	0.3569	0.0662	0.8000

1=3666* stems/ha (1.6m spacing)
2=3000 stems/ha (1.8m spacing)
3=1733 stems/ha (2.4m spacing)
4= 625 stems/ha (4.0m spacing)

* Figures exclude mortality up to 5m top height.

TABLE 8

Regression coefficients of the Richard Function Parameters b, k and m on respective Log Density for each Crop Characteristic.

Growth Characteristics	Growth Paras.	Regression coefficients b_0	b_1	b_2	b_3
Vol/ha	b	–251.67194	237.91143	–74.295295	7.6937755
	k	0.9831683	–1.1445837	0.4509529	–0.0568714
	m	98.721554	–93.204607	29.44918	–3.0901535
BA/ha	b	–60.673875	54.885367	–15.983722	1.5175977
	k	33.356622	–31.458282	9.8719758	–1.0280109
	m	180.27699	–168.12044	52.203013	–5.381726
Mean DBH/ha	b	61.074745	–59.466576	19.25527	–2.0579483
	k	–8.8622199	8.9257488	–2.9546853	0.3235010
	m	–48.388036	49.405919	–16.408052	1.7990441

TABLE 9

Biomathemathical Growth Equations for the calculation of "adjusted" values of the Richards Function Parameters (A, b, k and m) for each Growth Characteristic at a range of initial espacements.

Growth Characteristics	Growth Parameters	Equations
Vol/ha	A	$1585.5625+0.000002(203.90686)^x$
	b	$-251.67194+237.91143X-74.295295X^2+7.6937755X^3$
	k	$0.9831683-1.1445837X+0.4509529X^2-0.0568714X^3$
	m	$98.721554-93.204607X+29.44918X^2-3.0901535X^3$
BA/ha	A	$68.5115+0.002(14.2892)^x$
	b	$-60.673875+54.885367X-15.983722X^2+1.5175977X^3$
	k	$33.356622-31.458282X+9.8719758X^2-1.0280109X^3$
	m	$180.27699-168.12044X+52.203013X^2-5.381726X^3$
Mean DBH/ha	A	$-56.0625+151.1494(0.8942)^x$
	b	$61.074745-59.466576X+19.25527X^2-2.0579483X^3$
	k	$-8.8622199+8.9257488X-2.9546853X^2+0.3235010X^3$
	m	$-48.388036+49.405919X-16.408052X^2+1.7990441X^3$

$X=\log_{10}$ of density

TABLE 10

Input data for YIELD. FOR. generated by RICHAD. FOR, BMD and CURVE. FOR.

1					
1585.5625	0.000002	203.90686	251.67194	237.91143	
74.295295	7.6937755	0.9831683	1.1445837	0.4509529	
0.0568714	98.721554	93.204607	29.44918	3.0901535	
68.5115	0.002	14.2898	60.673875	54.885367	Biomathematical
15.983722	1.5175977	33.356622	31.458282	9.8719758	Growth
1.0280109	180.27699	168.12044	52.203013	5.381726	Equations
56.0625	151.1494	0.8942	61.074745	59.466576	
19.25527	2.0579483	8.8622199	8.9257488	2.9546853	
0.323501	48.388036	49.405919	16.408052	1.7990441	

3000.0 (Initial stocking density)

A	k	m			
26.4767	0.0319	0.5300		24.0	
30.2367	0.0306	0.4900		28.0	
40.6958	0.0197	0.3100		32.0	
39.8395	0.0243	0.3500		36.0	
37.7746	0.0351	0.5000	Top height/age growth models	40.0	Age
39.0250	0.0344	0.4500		44.0	
40.3514	0.0362	0.4500		48.0	
41.9092	0.0375	0.4500		52.0	
42.4380	0.0413	0.4800		56.0	
42.8787	0.0459	0.5200		60.0	
43.4904	0.0547	0.5600			

1333.6367 2319.8584 0.9391 (Surviving stems equation).

TABLE 11

Adjusted values for A, b, k and m Parameters for the different initial stocking densities (output of 'Forest').

A, B, K, M = parameters for Vol/ha
AA, BB, KK, MM = parameters for Ba/ha
AAA, BBB, KKK, MMM = parameters for mean DBH
AX = Initian stocking densities at 5.0m top height.

AX	A	B	K	M
5000	2283.0703	1.2064	0.0412	0.5005
3666	1926.1977	0.8380	0.0573	0.7144
3000	1799.9525	0.7611	0.0646	0.7803
2350	1707.5405	0.7866	0.0707	0.8047
1733	1645.9316	0.9227	0.0743	0.7810
1100	1606.6935	1.1232	0.0734	0.7211
770	1594.8347	1.0749	0.0689	0.7354
625	1591.2895	0.8870	0.0652	0.7992
AX	AA	BB	KK	MM
5000	105.9602	0.4568	0.0371	0.2930
3666	94.6729	0.6126	0.0957	0.5510
3000	89.2700	0.7194	0.1110	0.6096
2350	84.1682	0.8457	0.1125	0.6024
1733	79.5250	0.9819	0.0987	0.5278
1100	75.0266	1.0983	0.0750	0.4336
770	72.8267	1.0764	0.0808	0.5163
625	71.9026	1.0026	0.1043	0.6816
AX	AAA	BBB	KKK	MMM
5000	43.8833	0.4133	0.0993	0.9127
3666	45.4011	0.5542	0.0635	0.7214
3000	46.3938	0.5901	0.0503	0.6541
2350	47.6162	0.5894	0.0428	0.6198
1733	49.1611	0.5412	0.0431	0.6318
1100	51.5097	0.4297	0.0545	0.7118
770	53.3893	0.3635	0.0641	0.7792
625	54.5040	0.3538	0.0667	0.8027

TABLE 12

Yield Tables for an initial stocking density of 3000 stems/ha.

AX	A	B	K	M	
3000	1799.9525	0.7611	0.0646	0.7803	(1) = Vol/ha/top height equation
AX	AA	BB	KK	MM	
3000	89.2700	0.7194	0.1110	0.6096	(2) =Ba/ha/top height equation
AX	AAA	BBB	KKK	MMM	
3000	46.3938	0.5901	0.0503	0.6541	(3) = Mean DBH/top height equation

Stems	Age	TopHt	DBH	BA	Mean Volu.	Volume	MAI
2828	24.0	7.0	9.8	31.9	0.03	88.4	3.7
2681	28.0	8.6	11.5	39.0	0.05	133.4	4.8
2553	32.0	10.2	13.2	45.5	0.07	185.9	5.8
2441	36.0	11.8	14.8	51.2	0.10	243.1	6.8
2346	40.0	13.2	16.3	56.0	0.13	302.2	7.6
2264	44.0	14.5	17.7	60.0	0.16	360.8	8.2
2195	48.0	15.8	18.9	63.4	0.19	417.3	8.7
2136	52.0	16.9	20.0	66.2	0.22	470.4	9.0
2085	56.0	17.9	21.0	68.5	0.25	519.5	9.3
2043	60.0	18.9	21.9	70.4	0.28	564.3	9.4

Yield table for the first top height age model

(A = 26.4767)
(B = 1.0000)
(K = 0.0319)
(M = 0.5300)

(Table 10)