

Dimensional Control in the Sawmill

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SUMMARY

Following an outline of the theory of statistical control the construction of control charts for lumber dimension is explained with the aid of data.

INTRODUCTION

Just as variation is accepted in nature in general, so also does it exist in every manufacturing process, even the most precise. No two items are *exactly* alike. In the course of lumber production, variation in input factors (i.e. the material, the saw unit, the saw operator, the work method, etc.) will cause some variation in the dimensions of the output (the board, batten or plank). Thus a Mill sawing battens to a target 7 by 2 inches will produce material some of which will vary a few thousands of an inch from the target 2 inch thickness. This is fully acceptable to management and may be established as a standard for good sawing. Similar standards will be established for other dimensions. One would expect this small variation to remain relatively stable over time unless something happens to create a change in at least one of the input factors. The decision problem facing the sawmill manager is whether an observed difference between the actual measurement and the established standard is acceptable or is an exception to normal performance and necessitates corrective action. Shutting down the Mill unnecessarily can be costly, while the production of poorly sawn or over-dimensioned timber is wasteful and economically unattractive. How does management decide whether the production process is performing satisfactorily and what to do if it is not? The answer lies in statistical control whereby changes in the process, other than those due to chance causes, may be made known and action taken.

STATISTICAL CONTROL

The objective of control is to ensure that the manufactured product of future output conforms to some current or past standard of specification. This implies that variation in the product will remain

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stable and can be predicted. The underlying assumption is that stability in variation in the product will result from stability in the variation both within and between input factors but that a change in the variation contributed by the input factors will cause a change in the variation in the product. Thus the total variation in the product can be attributed to two possible causes (Bowman and Fetter, 1961)

- (i) small random fluctuations inherent in the process;
- (ii) identifiable causes, in particular something having gone wrong with the plant, the material or the method of control, causing either a shift in the average dimension of the product or a sudden increase in the variability.

The basis of control is to set limits to the amount of variation that can be ascribed to (i) above and to use this as a criterion for deciding at any given time whether a set of observed values indicate that an assignable cause of variation is present.

For the purpose of predicting the quality of future output it is necessary to select some method of describing the current output. The conventional statistics are the mean and standard deviation of the distribution which characterises the production process. These can be estimated as—

$$\bar{x} = \frac{\sum x}{n}$$

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

and

where x is the value of a variable, n is the number of values in the sample and \bar{x} and s are estimates of the population mean, μ , and standard deviation, σ , respectively. If the variable is continuous and the frequency distribution of measurements follow the well known bell-shaped or normal distribution, then definite areas are included within known standard deviations on either side of the mean as follows:—

- | | | | | | | |
|-------|-------------------|----------|-------|---------------------|-------------|------------------|
| (i) | $\mu \pm \sigma$ | includes | 68.26 | per cent | of the area | under the curve. |
| (ii) | $\mu \pm 2\sigma$ | „ | 95.45 | „ „ „ „ „ „ „ „ „ „ | | |
| (iii) | $\mu \pm 3\sigma$ | „ | 99.73 | „ „ „ „ „ „ „ „ „ „ | | |
| (iv) | $\mu \pm 4\sigma$ | „ | 99.99 | „ „ „ „ „ „ „ „ „ „ | | |

Given the mean and the standard deviation of a normal population of measurements it is possible to predict the proportion of

observations which can be expected to occur within given limits. Provided the proportion of observations *outside* the limits does not exceed expectation one may infer that the production process is in control at the given limits. This is the basis of the control chart technique.

CONTROL CHARTS

A sawmill producing battens 2 inches in thickness provides the data shown in columns 2 to 5 in Table 1. The measurements are given in the order of sawing and the sequence is obtained by reading across the rows, i.e. the first measurement is 1.95 the second is 1.98, etc.

TABLE 1

Batten No.	Batten thickness in inches				\bar{x}	s	R
1	1.95	1.98	1.98	2.00	1.978	.0206	.05
2	1.99	1.99	1.98	1.98	1.985	.0057	.01
3	1.97	1.99	1.96	1.97	1.972	.0126	.03
4	1.98	1.99	2.00	1.96	1.982	.0171	.04
5	1.99	1.98	1.95	1.99	1.978	.0189	.04
6	1.90	1.92	1.92	1.95	1.922	.0206	.05
7	2.11	2.01	2.02	2.02	2.040	.0469	.10
8	1.98	1.99	1.98	1.95	1.975	.0173	.04
9	2.02	2.05	2.00	2.02	2.022	.0206	.05
10	2.00	2.00	2.00	2.02	2.005	.0100	.02
11	2.01	2.00	2.00	2.05	2.015	.0238	.04
12	2.00	2.00	2.02	2.04	2.015	.0191	.04
13	2.00	2.02	2.02	2.03	2.018	.0126	.03
14	2.09	2.04	2.06	2.08	2.068	.0222	.05
15	1.98	2.00	2.01	2.02	2.002	.0171	.04
16	2.00	2.00	2.00	2.04	2.010	.0200	.04
17	2.00	2.01	2.00	2.00	2.002	.0050	.01
18	2.00	2.00	2.00	2.02	2.005	.0100	.02
19	2.04	2.00	2.00	2.00	2.010	.0200	.04
20	2.03	2.02	2.00	1.98	2.008	.0222	.05
21	2.00	2.00	2.02	2.05	2.018	.0050	.05
22	2.02	2.00	2.00	1.99	2.002	.0126	.03
23	1.96	2.00	2.00	2.00	1.990	.0200	.04
24	2.00	2.00	2.05	2.02	2.018	.0236	.05
25	2.04	2.02	2.00	2.00	2.015	.0191	.04

Using the above equations, estimates of the population mean and standard deviation¹ are obtained as—

$$\bar{x} = 2.0022 \text{ inches and } s = 0.0318 \text{ inches.}$$

The data, when summarised in the form of a frequency distribution (Table 2) is approximately symmetrical about the mean and may be assumed normal.

TABLE 2

Class interval	Frequency
1.90—1.91	1
1.92—1.93	2
1.94—1.95	4
1.96—1.97	5
1.98—1.99	18
2.00—2.01	40
2.02—2.03	17
2.04—2.05	9
2.06—2.07	1
2.08—2.09	2
2.10—2.11	1

¹It is more convenient to use the equation

$$s = \sqrt{\frac{\sum x^2 - (\sum x)^2/n}{n-1}}$$

when a calculator is available.

A knowledge of the form of the distribution, although useful, is not essential. If the measurements in Table 1 are divided into subgroups of, say 4, the distribution of the mean values of the subgroups will tend towards normality, with standard error (standard deviation of the distribution of means) $s_{\bar{x}}$ equal to s/\sqrt{n} , where n is the size of each subgroup. Furthermore, the standard deviation of each subgroup may be calculated and the values arranged in a frequency distribution which will have a standard error (standard deviation of the distribution of sample standard deviations) s_s approximately equal to $s/\sqrt{2n}$, where n is the subgroup size. This follows from the Central Limit Theorem (Mendenhall 1967), which states that,

sample estimates of means or totals tend towards a normal distribution, regardless of the distribution of the parent population. The tendency towards normality is more pronounced as sample size increases. For control chart purposes, however, a subgroup or sample size of 4, 5 or 6 is most common for reasons which will be stated later.

If each row in Table 1 is treated as a subgroup of four, control may be established based on the statistics which result from computing the average and standard deviation of the distribution of means of samples of four. An estimate of the standard error is obtained from $s_{\bar{x}} = s/\sqrt{n} = 0.0318/\sqrt{4} = 0.0159$ and $\bar{\bar{x}}$ is equal to the mean of the subgroup means. A chart may then be prepared with a central line corresponding to $\bar{\bar{x}}$ and limit lines above and below corresponding to given standard errors (Figure 1).

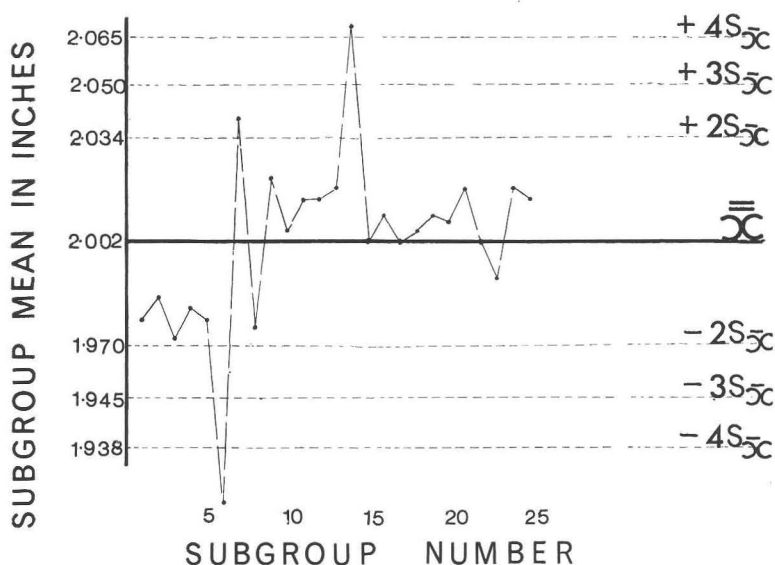


Fig. 1

Control chart for averages.

For a normal distribution expectation is 4.55 per cent outside $\bar{\bar{x}} \pm 2s_{\bar{x}}$, 0.27 per cent outside $\bar{\bar{x}} \pm 3s_{\bar{x}}$ and 0.1 per cent outside $\bar{\bar{x}} \pm 4s_{\bar{x}}$. If the subgroup means in Table 1 (column 6) are plotted on a time basis it will be seen that observed values exceed expectation suggesting that some assignable causes of variation are present causing a shift in average thickness.

The purpose of the control chart for averages is to detect a shift in the average, the assumption being that variation remains constant. Yet, to take an extreme case, four thickness measurements might read 1.0", 1.5", 2.5", 3.0", giving an average thickness of 2.0" which would fall on the centre line in Figure 1. Even though the average thickness is correct, such a product would of course be unacceptable. In order to exercise greater control over the production process, it is necessary to have a chart which will show changes in variation.

An estimate of the standard deviation of the sample standard deviations for subgroups may be approximated by $s_g = s/\sqrt{2n}$. In the above example

$$s_g = 0.0318/\sqrt{8} = 0.0112$$

Alternatively, the standard deviation of each subgroup may be calculated and an estimate of the standard deviation, \bar{s} , obtained by adding the subgroup standard deviations and dividing by the number of subgroups. In the above example

$$\bar{s} = 0.4426/25 = 0.0178$$

This new estimate of the population standard deviation is smaller than that obtained previously from all 100 measurements. The reduction may be attributed to the fact that the first estimate (0.0318) included variation due to the pooling of data from populations with different means.

Taking \bar{s} as an estimate of the population standard deviation

$$s_g = 0.0178/\sqrt{8} = 0.0063$$

A control chart for standard deviation may be set up with \bar{s} as the centre line and control limits set as $\bar{s} \pm 3s_g$ (Figure 2).

The standard deviation of each subgroup when plotted on a time basis will show where variation within subgroups is excessive. Usually, the setting of control limits does not involve the calculations of the standard deviation for each subgroup. When production is the effect of a single generating process the standard deviation of the single measurements will provide a good estimate of the population standard deviation and this can be used in the equation $s_g = s/\sqrt{2n}$ to obtain control limits. However, the standard deviation of each subgroup must be known if it is proposed to plot their respective values. In any event the approach demands a certain amount of tedious calculation and it is this difficulty which makes the use of a standard deviation chart somewhat impractical in the mill. For this reason the *range*, which is defined as the difference between the

highest and lowest measurement within the subgroup, is commonly used as a measure of variation in the construction of control charts. Range is an efficient estimator of standard deviation only when the subgroup size is small. This and the fact that there is generally less variability within subgroups than between subgroups when their size is small, are the reasons for selecting a relatively small subgroup size.

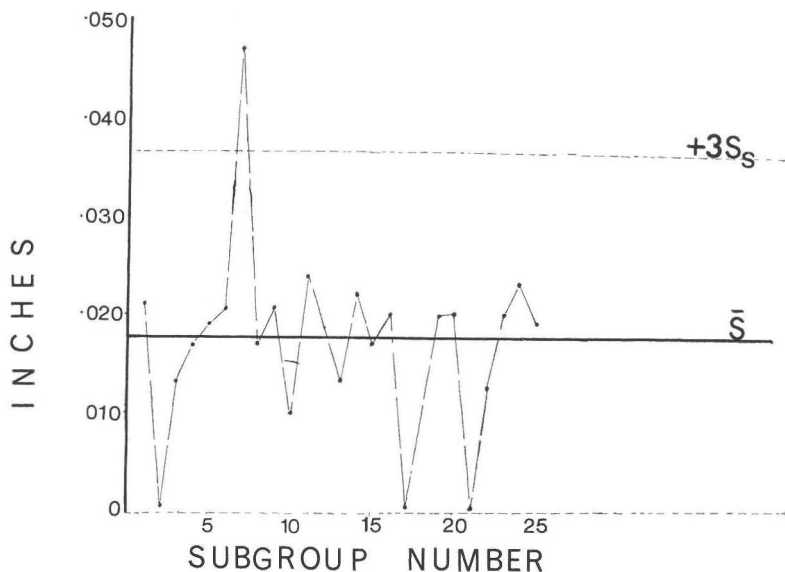


Fig. 2 *Control chart for standard deviation.*

CONTROL CHARTS USING RANGE:

To provide a practical example of the use of Range in setting up control charts, it is assumed that four thickness measurements are taken on each of 25 battens sawn to a target thickness of 2 inches. The average thickness, \bar{x} and range, R are recorded for each subgroup of four measurements (Table 1).

The mean of the subgroup averages

$$\bar{\bar{x}} = \Sigma \bar{x} / n = 50.055 / 25 = 2.0022$$

and average range

$$\bar{R} = \Sigma R / n = 1.00 / 25 = 0.04$$

are calculated.

CONTROL CHART FOR AVERAGES

In order to use the range for control chart purposes it is necessary that the relationship between range and standard deviation should be determined. This has been done by Pearson (1932, 1941-42) who has tabulated the ratio of range to the population standard deviation, $w = R/\sigma$. An estimate of the population standard deviation may be obtained by multiplying the range in a single sample or the mean range in a number of samples by a factor. In practice the best available estimate of the expected range is \bar{R} , the average of range values for all subgroups, so an estimate of σ is obtained as

$$s = \bar{R}/d$$

where $1/d$ is the multiplication factor.

Since $s\bar{x} = s/\sqrt{n}$ a control chart for averages may be set up with upper confidence limits, $\bar{x} + 3R/d\sqrt{n}$ and lower confidence limits, $\bar{x} - 3R/d\sqrt{n}$.

To simplify calculation let

$$A = 3/d\sqrt{n}$$

then the control chart limits become

$$\bar{x} \pm A\bar{R}$$

Values of A , abstracted from Brown and Bethel (1958), are tabulated for different subgroup sizes, n in Table 3.

In the above example the value of A is 0.73 for subgroup size 4, $\bar{x} = 2.0022$ and $\bar{R} = 0.04$. Therefore, the control limits are $2.0022 \pm 0.73(0.04) = 2.0022 \pm 0.0292$, giving

$$UCL_{\bar{x}} = 2.0314$$

and

$$LCL_{\bar{x}} = 1.9730$$

A control chart for averages is constructed (Figure 3). The chart is the same as that in Figure 2 except that the confidence interval is smaller. However, if the mean of the subgroup standard deviations had been used to construct control limits in Figure 2, both figures would be almost identical. Range is therefore considered to be an efficient estimator of the population standard deviation in this instance.

The means of successive subgroups are now plotted on the chart. According to probability theory the expectation of a mean falling outside the control limits is 0.27 per cent. In the data, the

means of battens number 6, 7 and 14 fall outside the control limits suggesting that some attributable causes of variation are present.

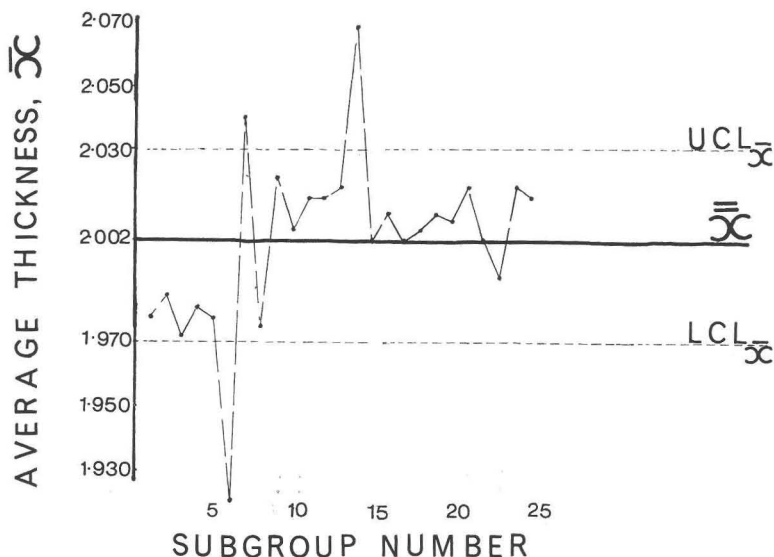


Fig. 3

Control chart for averages.

Possibly the saw operator made an incorrect setting or the variation may be due to a looseness in the setworks coming into play occasionally. The manager must use his judgement whether to take action at this stage. His reasoning will be conditioned by the attitude of the buyer towards the material. Will it be acceptable? At the same time he will be concerned that he is getting the maximum lumber yield from his logs. If the specified average thickness is 2 inches and the material being sawn has an average thickness of 2.1 inches there is an unnecessary wastage of 5 per cent. It may be argued that the dimensions of battens number 6, 7 and 14 in the above data are quite acceptable. This may well be true but the fact remains that the mill is not performing as well as it is capable of doing and it is wasteful to run any machine at a standard below its potential.

TABLE 3:

n	Factors for \bar{x} chart	Factors for R chart	
	A	D_3	D_4
2	1.88	0	3.27
3	1.02	0	2.57
4	0.73	0	2.28
5	0.58	0	2.11
6	0.48	0	2.00
7	0.42	0.08	1.92
8	0.37	0.14	1.86
9	0.34	0.18	1.82
10	0.31	0.22	1.78

CONTROL CHART FOR RANGE

The control chart for averages will show if a shift occurs in the average thickness, the assumption being that variation remains more or less constant. The same value for average thickness could conceal a shift in the variability of the individual measurements but if the variability increases it is probable that, with continued sampling, an average outside the control limits will be obtained. Reliance on the control chart for averages to detect an increase in variation is, however, an inefficient method of control. It is preferable to have a chart which will show changes in variation. Range, as a measure of variation, is used again.

To prepare a control chart for range it is necessary to have available an estimate of the standard deviation of the population of ranges. This can be developed in the following way (Bowman and Fetter 1961)

$$w = R/\sigma \text{ and } R = w\sigma$$

Since the standard deviation of a distribution of values of w is $\sigma_w\sigma$, the standard deviation of values of R is $\sigma R = \sigma_w\sigma$. σR may be estimated from a sample as sR and, similarly, σ as s . Then $sR = \sigma_w s = \sigma_w R/d$. Since $s = R/d$. Control limits for range can be set at

$$\bar{R} \pm 3 \sigma_w \bar{R}/d.$$

To simplify the procedure let

$$D_4 = 1 + 3 \sigma_w/d$$

and

$$D_3 = 1 - 3 \sigma_w/d$$

Then the control limits for the range chart are

$$UCL_R = D_4 \bar{R}$$

$$LCL_R = D_3 \bar{R}$$

For different sample sizes, σ_w , the standard deviation of the values of R/σ , has been tabulated (Pearson, 1932) and values of D_4 and D_3 abstracted from Brown and Bethel (1958) are given in Table 3.

In the example given the values of D_4 and D_3 for $n=4$ are 2.28 and 0 respectively. The control limits are

$$UCL_R = 2.28 (.04) = 0.0912$$

$$LCL_R = 0 (.04) = 0$$

A control chart for range showing the mean range and the upper and lower control limits is then constructed (Figure 4) and the range for each subgroup (R column in Table 1) plotted on the chart.

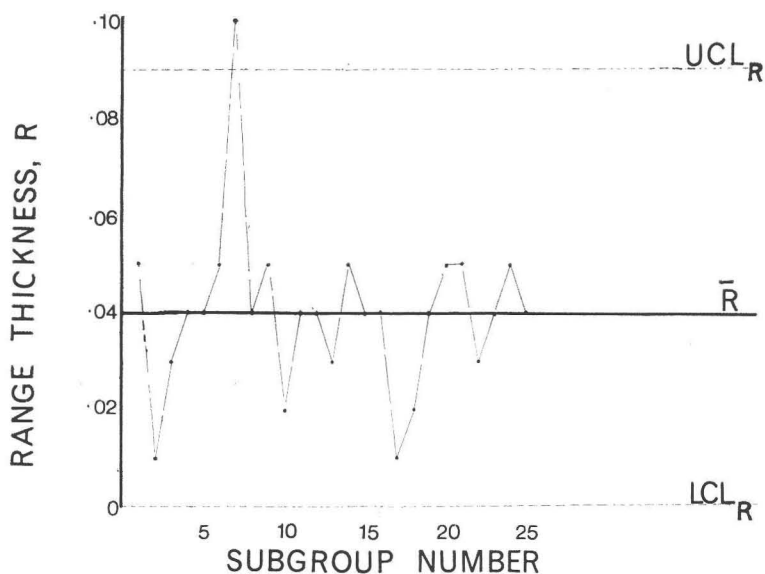


Fig. 4

Control chart for range.

It is apparent from the chart that variation in thickness within each batten is in a fairly good state of control. The one exception is batten number 7 whose high range value may be due to a slight movement in the log as the saw entered, possibly as a result of inadequate dogging.

The use of control charts as an effective means of control depend on their ability to show up attributable causes of variation in the production process. When a reading falls outside the control limits it may be due to some attributable cause of variation or it may be due to random causes. However, the probability of readings falling outside the control limits, due to random causes, can be determined in advance and used as a basis for deciding whether or not to take action. If control limits are set at ± 3 standard errors and action is taken whenever a reading falls outside the limits, there are about 3 chances in 1,000 that a wrong decision will be made. This is the probability of mistakenly inferring that the process is out of control when it is really in control. If ± 2 standard errors are used as control limits on the same basis there are about 45 chances in 1,000 of mistakenly inferring that the process is out of control when nothing is wrong. Obviously, wider limits decrease the probability of looking for trouble in the process when there is none. On the other hand, wider limits decrease the probability of detecting trouble in the process when it exists, but the probability of mistakenly inferring that the process is in control when it really is not depends on the magnitude of the shift that has occurred.

The selection of control limits should, if possible, depend on some consideration of the costs involved. These include the cost of investigating trouble in the process when it does not exist, and the cost, in the form of defective output, of not investigating trouble when it does exist. When ± 3 standard errors are used (as in the above example) a certain amount of judgement must be used in making decisions based on observations falling outside the control limits.

The procedure for setting up a control plan for dimensional control in the sawmill, described above, has wide application and can be applied to the control of any quality which can be measured on a continuous scale. The thickness of fibreboard and chipboard are two wood using processes which also lend themselves to this type of control.

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