The development of a taper model and a diameter at breast height to total height model to predict user defined roundwood assortment volumes in Sitka spruce first thinning plantations in Ireland

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Abstract
A taper equation and a diameter at breast height (DBH) to total height model were developed for Sitka spruce (*Picea sitchensis* (Bong.) Carr.) trees of first thinning stage in Ireland. The use of the models may be of benefit to forest practitioners who require information on the volumes of specific assortments prior to harvest, particularly where the practitioners wish to define the dimensions of these assortments themselves, such as in the wood energy sector. The models could be implemented into a tool for practitioners, once further field trials are undertaken to estimate the degree to which errors are propagated. To develop the models, data were collected from 433 trees on five private forest sites in Ireland. The Kozak (2004) variable exponent taper equation was parameterised using the data. This taper equation utilises total height as an input, and therefore an estimate of total height per tree was required. A Chapman Richards equation was parameterised to predict total height from DBH. Using the equations together, the volumes of 90 sample trees were predicted, and then compared to actual measurements. The equations predicted the full stem length volume with a standard error estimate of 0.0098 m^3 per tree, and a bias of 0.00003 m^3 per tree.

Keywords: *Kozak's taper model, Chapman Richards height model, forest energy, volume estimation.*

Introduction

The National Forest Inventory (NFI) (2007) identified 233,000 ha of Sitka spruce (*Picea sitchensis* (Bong.) Carr.) under 20 years of age in Ireland and further identified that 120,000 ha of this species was in private ownership. Philips (2011) estimated that total annual thinning area will more than double from the current 22,800 ha to 49,400 ha by 2028, as this young forest resource comes into production. He noted that the majority of the increase in annual roundwood production over the coming years will come from the private sector forests and warned that without first thinning, roundwood production will be much lower than predicted. Similarly, forecast models predict that the demand for wood biomass for energy will increase from 1.589 million $m^3\ yr^{-1}$ in 2011, to 3.084 million m^3 in 2020 (CRDG 2011). It was noted that expansion of the wood energy sector may cause competition for the panel board industry seeking wood fibre, so supply and demand will remain finely balanced.

Standard forest thinning control and pre-sale measurement practices estimate standing tree volume to merchantable timber height. Merchantable timber height is

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where the stem tapers below 7 cm diameter (Matthews and Mackie 2006). The actual volume, as cut by a harvesting machine, will differ from this estimate in two ways. The volume will be less when cutting to a specified length, as only full log lengths can be processed. For example, a stem of 4.5 m merchantable volume height will only produce one 3 m length log, the other 1.5 m will be discarded. The volume will be greater than the estimate when using a full stem assortment, as the full stem includes the material above the height where the stems taper below 7 cm diameter (Keogh 1987). In the first thinning of a Sitka spruce stand, an average of 26% additional biomass was recovered in energywood harvesting, where the whole stem was processed into variable lengths, compared to harvesting standard roundwood assortments (Kent et al. 2011).

Research into the taper of forest trees is documented as far back as 1913 (Stoehr 1955), but perhaps some of the earliest functions constructed for prediction purposes was from work carried out in British Colombia (Newnham 1958). By 1969, it was well recognised that a total volume per ha estimate was no longer sufficient for harvest planning. It was necessary to be able to estimate the volume of specific log sizes and also the number of such logs that could be produced from a growing forest. At first, it was thought that additional upper stem measurements would need to be taken during inventory fieldwork. However, after the development and testing of a number of taper equations, it was shown that stem modelling could produce accurate results without the need for upper stem measurements (Kozak et al. 1969). Since then taper functions have been used across the globe for predicting upper stem diameters and log volumes. They are frequently used in inventory and growth modelling projection systems due to their flexibility and ability to estimate multi product volumes (Trincado and Burkhart 2006). More recently, Fonweban et al. (2011) developed taper equations for Scots pine (*Pinus sylvestris* L.) and Sitka spruce in Northern Britain. In Ireland, Nieuwenhuis et al. (2005) used a taper equation as part of a value-maximisation decision-support tool in a sawmill production chain.

Taper is defined as "the rate of narrowing in diameter in relation to increase in height of a given shape" (Gray 1956). Different taper equations use different shapes to describe a tree's stem: the lower section near the butt being a frustum of a neiloid, the middle section being a frustum of a paraboloid, and the top section being a paraboloid (Avery and Burkhart 1983). Many of the more complex taper models use a number of polynomial functions joined together to recreate the stem form (Trincado and Burkhart 2006). Some models will fit specific tree species and conditions better than others, so it is beneficial to test a number of models with a particular type of forest dataset to find the best fit, as per Walters and Hann (1986). According to Kozak (2004), taper equations are superior to volume equations, as volume equations only estimate total or merchantable volume, whereas taper equations provide estimates of: "i) diameter at any point along the stem, ii) total stem volume, iii) merchantable volume to any top diameter, iv) merchantable height to any top diameter, v) individual volumes for logs of any length at any height from the ground."

The aim of this study was to develop a system of models which can be used to predict stem volumes in Sitka spruce first thinnings. This may be of particular benefit to private forest managers in the planning phase of thinning operations. The private forest sector has not yet developed long-term contracts to supply small roundwood to end-user defined specifications. The models could potentially assist the manager in optimising the value of thinnings through market selection of the timber before harvesting. The models could be used by both state and private sectors to simulate the harvest volume from user-specified assortment dimensions. Harvesting methods could then be selected that best match the optimal return from the forest.

Materials and methods

Data collection

The data were collected on five sites; three sites in the west of Ireland, one in the midlands and one in the south (Table 1). The ages of the stands ranged from 13 to 20 years, and were even-aged Sitka spruce monocultures, except for one site where the spruce was intimately mixed with Japanese larch (*Larix kaempferi* L.). The Japanese larch was not used in this study. In order to evaluate the performance of the models, some of the data were partitioned to be used for independent validation. This method of evaluation is described by Kozak and Smith (1993), and was used in a similar context in a study by Nieuwenhuis et al. (1999) where a taper model and diameter at breast height (DBH) to height model were also developed for volume estimation.

Table 1 details the site descriptions and the number of sample trees measured during this study. The models were developed with the data from sites 1-4, and the data from site 5 was used for validation. In total, 433 sample trees were felled and measured. Approximately 30 trees were selected per line in each stand. The lines were picked at random from each site. The sample trees were felled by chainsaw and measured for total height and DBH. The stem was marked at 1 m intervals from the base to the tip, and the diameter at the mid-diameter point of each interval recorded to the nearest cm (rounded down). Figure 1 displays a scatterplot of data showing relative height versus relative diameter. The horizontal line of data points appearing at a relative diameter value of 1.0 is the result of every tree being measured for its DBH, as relative diameter was taken as the quotient between the upper stem diameter and the DBH of the tree. The distortion apparent above 1.0 relative diameter is the result of the buttressing of the stems close to the ground.

Table 1: *Characteristics of the sampling sites.*

Site	Location	Area (ha)	Age (years)	Stocking (stems ha^{-1})	Mean DBH (cm)	Top height (m)	No. of sample trees
1	Abbyfeale, Co. Limerick	10	20	2,191	17	13.5	90
2	Ballybofey, Co. Donegal	21	13	2,455	14	11.2	75
3	Bweeng, Co. Cork	10	17	2,252	13	11.1	88
4	Toormakeady, Co. Sligo	14	16	2,624	13	10.9	90
5	Woodberry, Co. Galway	27	17	2,199	15	12.3	90

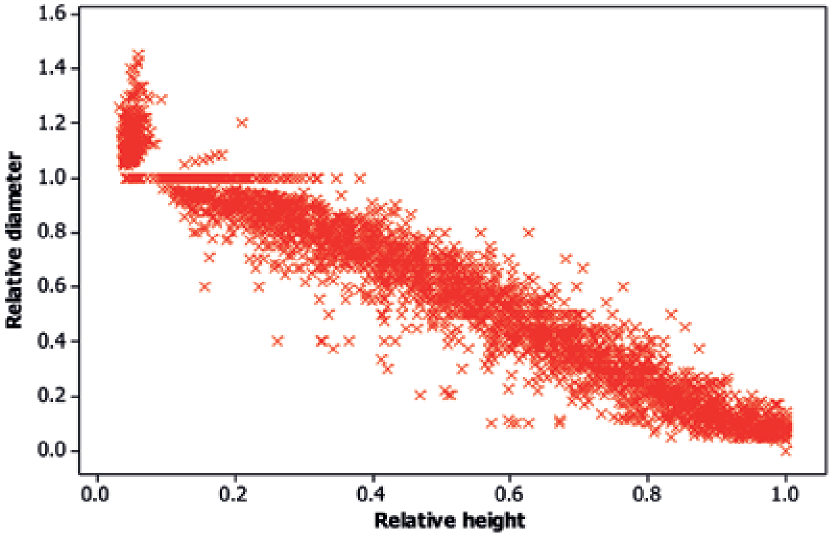

Figure 1: *Scatter plot of relative diameter (the upper stem diameter divided by the DBH of the tree) versus relative height (the height of the upper stem diameter divided by total height of the tree) of the data collected.*

Taper equation

A taper equation formulated by Kozak (2004) was parameterised using the data collected. Kozak's taper equation was chosen as it has been used extensively in modelling the taper of plantation trees, including studies based in Ireland and the UK (Fonweban et al. 2011, Nieuwenhuis et al. 1999). The Minitab 16 statistical software package (Minitab Inc. 2010) was used to fit the equation using non-linear regression. Kozak's taper equation is given as Equation 1:

$$d=\beta_1 DBH^{\beta_2} x^{[\beta_3 z^2+\beta_4 \ln(z+0.001)+\beta_5 (\frac{DBH}{ht})]} \quad (1)$$

where d is predicted diameter (cm), x is $\frac{1-\sqrt{z}}{1-\sqrt{p}}$, p = point of inflection is $\frac{1.3}{ht}$, ht is total tree height (m), h is height along the stem at predicted diameter (m), z is $\frac{h}{ht}$ is relative height of predicted diameter, DBH is diameter at breast height (cm), β_1 to β_5 are the parameters to be estimated from the regression analysis (Fonweban et al. 2011, Kozak 2004).

DBH to total height equations

As the taper equation utilises total height as an input, an estimate of total height per tree was required. A Chapman Richards DBH to total height model, as cited by Kershaw et al. (2008), was parameterised using the data. Recently, this Chapman Richards function has been used to predict tree height by the NFI (2007). The NFI (2007) fitted this model to Sitka spruce data taken from plots throughout Ireland. In this study the model was developed in the same way: the model was parameterised using the data from sites 1 to 4. When using the model for predicting the total height of individual trees in a plot, the height and DBH of a sample height tree was used to localise the model to this plot. This was deemed as a necessary inclusion as the DBH to height relationship is not homogenous across different site conditions (Diéguez -Aranda et al. 2006). It should be noted that the NFI (2007) locked β_3 at 0.7 in 84% of the cases in which the Chapman Richards model was used. This methodology was also tested in this study. The Chapman Richards model is given as:

$$ht = 1.3 + \beta_1(1 - e^{-\beta_2 dbh})^{\frac{1}{\beta_3}} \quad (2)$$

where ht = total height of the tree, and β_1 to β_3 are the parameters to be estimated from the regression analysis (Kershaw et al. 2008).

Model evaluation

The models were evaluated for their ability to fit the data sets by assessing the root mean squared error (RMSE) from the output of the regression analysis, as per Fonbewan (2011). As recommended by Kozak and Smith (1993), the models were evaluated for their prediction abilities and were compared to validation data sets for bias and standard error estimates (SEE).

Average bias was defined as:

$$\text{Mean Bias} = \frac{\sum_{i=1}^{n} (Y_i - \hat{Y}_i)}{n} \quad (3)$$

where Y_i is actual observation, $\hat{Y}_i$ is predicted value of the actual observation, and n the number of observations.

The standard error estimate was given as:

$$\text{Standard Error Estimate} = \sqrt{\frac{\sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2}{n-k}} \quad (4)$$

where k is the number of estimated parameters (Jiang et al. 2005).

Results and discussion

Taper model

Table 2 details the parameter estimates for the taper model, and the associated 95% confidence interval for each estimate. The RMSE was estimated at 1.001 cm. Kozak's model was fitted to Sitka spruce in the UK with a RMSE of 0.983 cm (Fonweban et al. 2011), which is similar to the 1.001cm in this study. Figure 2 displays the residuals of the fit. Figure 3 displays a histogram of the residuals.

Table 2: *Coefficients and uncertainty ranges from regression analysis to parameterise.*

Parameter	Estimate	95% Confidence Interval	
β_1	1.14369	(1.10540,	1.18341)
β_2	1.00093	(0.98956,	1.01231)
β_3	-0.15975	(-0.20687,	-0.11306)
β_4	1.30694	(1.23812,	1.37624)
β_5	0.06093	(0.03453,	0.08752)

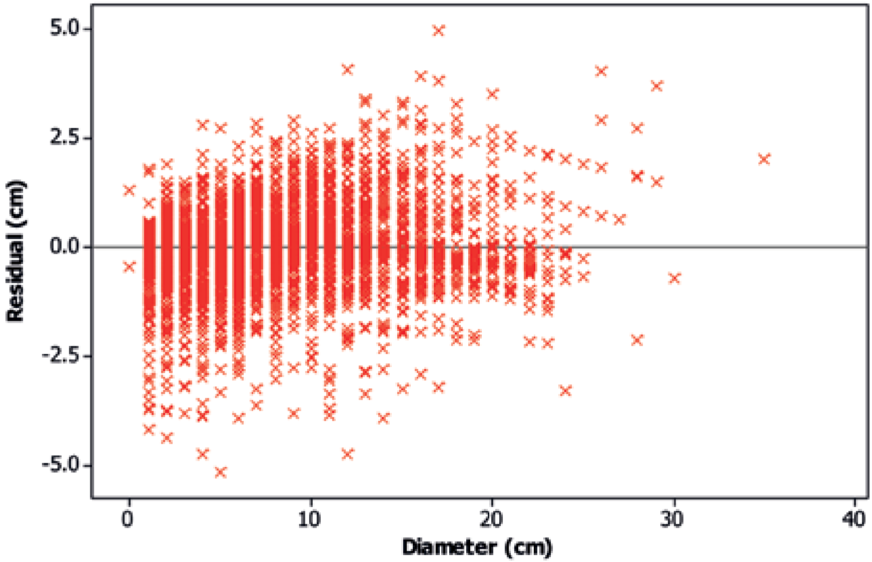

Figure 2: *Residuals from the fit of the taper equation to the dataset.*

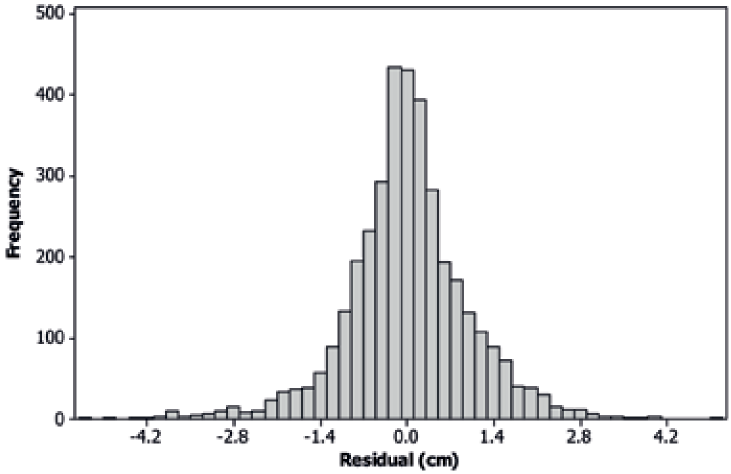

Figure 3: *Histogram showing the frequency distribution of the residuals from the fit of the taper equation to the dataset.*

To assess the model's prediction abilities, the model was used to estimate diameters at 1 m intervals from the validation data. The validation data comprised 90 trees, giving a total of 979 predictions. The results were then compared to the actual measured stem diameters of these trees as observed in the field. The input data to the models were the DBH and total height as measured for each tree. Table 3 provides the SEE and mean bias. The data are grouped by each 10% increment of relative height to enable comparisons along the stem. The overall SEE is 0.85 cm, and the overall bias is -0.06 cm.

Table 3: *Prediction statistics for the parameterised taper model using the validation data.*

Relative height %	n	SEE (cm)	Bias (cm)
0-10	99	1.07	0.09
10-20	168	0.53	-0.38
20-30	90	0.86	-0.13
30-40	93	0.83	0.08
40-50	91	0.83	0.38
50-60	88	0.84	0.34
60-70	90	0.83	-0.08
70-80	90	0.73	-0.22
80-90	89	0.60	-0.38
90-100	81	0.46	-0.10
Overall	979	0.85	-0.06

DBH to total height model

The parameter estimates and associated confidence intervals of the DBH to total height model are detailed in Table 4. Figure 4 displays the fitted model line through the data set, and Figure 5 presents a histogram of the residuals.

Table 4: *Parameter estimates for the DBH to total height model.*

Parameter	Estimate	95% Confidence Interval	
β_1	10.8599	(10.4688,	11.2950)
β_2	0.1558	(0.1427,	0.1703)
β_3	0.7000		(locked)

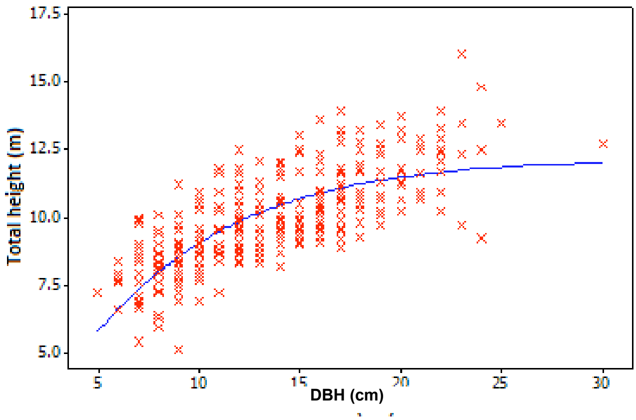

Figure 4: *Data with fitted DBH to total height model line.*

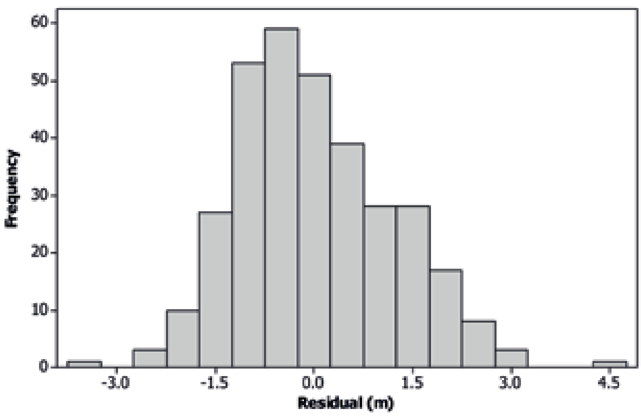

Figure 5: *Histogram of the fit residuals from the DBH to total height model.*

Table 5: *Statistics for parameterised DBH to height model using validation data.*

Plot	n	Top height (m)	QMDBH[a] (cm)	Dominant DBH (cm)	Model statistics	
					SEE (m)	Bias (m)
1	31	13.6	12.1	21	0.99	-0.67
2	29	9.7	13.0	22	0.60	0.91
3	30	13.7	15.6	23	0.92	-0.26

[a] QMDBH = quadratic mean DBH.

The validation data were collected from three plots at a single site at a single site (Table 5). The model was localised to each plot through non-linear regression as described in the methodology section.

The parameterised Chapman Richards model was localised by adjusting β_1. This was done using a sample height measurement and the associated DBH of the sample height tree. Ideally, a number of trees would be measured for height and used for the adjustment. However, it was found that there were two benefits to using only one tree:

i) it required no additional measurement;

ii) the adjusted parameter of β_1 could be defined mathematically without non-linear regression.

As the majority of forest managers may not have access to non-linear regression tools, it was important that the models could be used as a stand-alone entity that could be implemented into a simple spread-sheet software package. As all other terms in the equation are known, it was possible to rewrite the equation to solve for β_1. This gives the model the ability to localise to a plot by entering to the equation the values of sample tree height (sH) and DBH of the sample height tree (DBH_{sH}) measured in the stand, as outlined below:

$$\text{If total height (ht)} = 1.3 + \beta_1 \times (1 - e^{-0.155793DBH})^{\frac{1}{0.7}} \quad (5)$$

Then, using the sample height (sH) and the dbh of the sample height tree (DBH_{sH}), the equation was rewritten as follows to find β_1:

$$\beta_1 = (sH - 1.3)/(1 - e^{-0.155793DBH_{sH}})^{\frac{1}{0.7}} \quad (6)$$

And therefore, total height can be found from:

$$H = 1.3 + \left[(sH - 1.3)/(1 - e^{-0.155793DBH_{sH}})^{\frac{1}{0.7}}\right](1 - e^{-0.155793DBH})^{\frac{1}{0.7}} \quad (7)$$

Evaluation of the developed models for estimating stem volumes

The above equations can be used to predict the log assortment options available from a particular tree stem. This can be accomplished by predicting stem diameters at heights corresponding to the assortment lengths, and assessing whether the diameters are above the minimum top diameter threshold. All calculations can be made in a simple spread-sheet. Importantly, this method would only utilise data collected in a standard thinning control assessment, as described by Matthews and Mackie (2006).

However, the errors associated with each sub-model will contribute to a total combined error. This total error is the result of the output of the DBH to total height model (which has an error) being used as an input variable to the taper model (which also has its own error). Because multiple predictions are used to estimate the assortment volumes, these errors will also be propagated for each volume estimate. The methods for calculating the total error for using the two models together in this manner will require an extensive body of additional work. This was not within the scope of the present study.

However, an evaluation of the performance of the equations' ability to predict volumes was made. This was done by comparing predicted and actual volume results using the data from site 5. The stem volumes were estimated in 1 m sections from the base to the tip. The results in Table 6 show that residual error and bias were low. The overall SEE was 0.0098 m^3 per tree. The overall mean bias was small, at 0.00003 m^3. Figure 6 displays the predicted versus the measured stem volume of the validation data.

Table 6: *Statistics for the predicted volumes of the validation data.*

Plot	Top height (m)	QMDBH (cm)	Mean stem vol.[a] (m^3)	SEE $stem^{-1}$ (m^3)	Mean bias $stem^{-1}$ (m^3)	Predicted total vol. $plot^{-1}$ (m^3)	Measured total vol. $plot^{-1}$ (m^3)
1	13.60	12.10	0.06	0.0112	-0.003	1.84	1.74
2	9.70	13.00	0.06	0.0064	0.003	1.58	1.65
3	13.70	15.60	0.11	0.0102	0.001	3.18	3.20
Overall				0.0098	0.00003	6.59	6.59

[a] Measured.

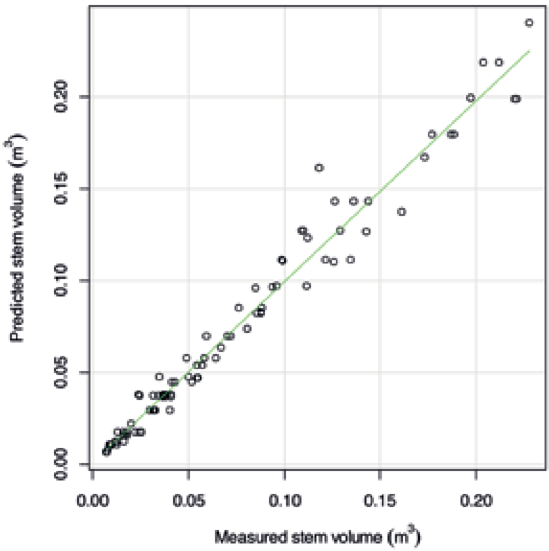

Figure 6: *Predicted versus measured stem volume (m³) of the validation data.*

Conclusions

For Sitka spruce first thinning plantations in Ireland, Kozak's taper equation has been parameterised to predict diameters along tree stems. To estimate the total height of trees, the Chapman Richards model was parameterised using the height and DBH for a single tree in each plot, and then applied to all trees in that plot. The taper function and DBH to height function could be implemented into a tool to predict the volume of different assortments in a stand prior to thinning. Importantly, it would not require any additional measurements beyond those normally taken in the thinning control measurement procedure. Overall, when compared to the validation data, the equations predicted the full stem length volume with a SEE of 0.0098 m^3 per tree, and a bias of 0.00003 m^3 per tree. These equations could be used for trees from 5 cm to 30 cm DBH, and for heights from 5.1 m to 16.0 m. With more data, the equations could be improved to predict outside these ranges.

In private forest holdings, the models developed in this study may help forest managers or owners plan the timing of first thinning, anticipate the harvesting resource capacity required, and may aid in the identification and marketing of specific product types to customers. Where a wood energy market is preferred, the equations could be used to estimate the additional volume recoverable by harvesting whole stems and cross-cutting in variable lengths.

However, further investigation is required on how the errors of the individual models will propagate to a total error. A trial of how the models perform in predicting the assortment volumes during a commercial first thinning should be undertaken. When fully validated, the models could be implemented into a simple spread-sheet program for distribution as a tool to practitioners. The practical implications of this work include:

- The ability for forest managers to predict the volumes of a variety of assortment options, where the dimensions of the assortments can be defined by the manager, and may assist in the marketing of timber from first thinnings prior to harvesting.
- The statistical models, comprising the basis of a practical tool, have been developed so that no additional measurements are required beyond the standard thinning control assessment procedure.
- The models will require field testing during commercial operations to fully evaluate their performance.

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